The Lattice Computing (LC) Paradigm*

Vassilis G. Kaburlasos[0000−0002−1639−0627]

International Hellenic Univertity (IHU),
Human-Machines Interaction Laboratory (HUMAIN-Lab),
Department of Computer Science, 65404 Kavala, Greece
vgkabs@teiemt.gr
http://humain-lab.teiemt.gr

Abstract. The notion of Cyber-Physical Systems (CPSs) has been introduced to account for technical devices with both sensing and reasoning abilities including a varying degree of autonomous behaviour. There is a need for supporting CPSs by mathematical models that involve both sensory data and cognitive data towards improving CPSs effectiveness during their interaction with humans. However, a widely acceptable mathematical modelling framework is currently missing. In the aforementioned context, the Lattice Computing (LC) paradigm is proposed for mathematical modelling in CPS applications based on lattice theory by unifying rigorously numerical data and non-numerical data; the latter data include (lattice ordered) logic values, sets, symbols, graphs and other. More specifically, the "cyber" components of a CPS involve non-numerical data, whereas the "physical" components of a CPS involve numerical data. A promising advantage of LC is its capacity to compute with semantics represented by a lattice (partial) order relation.

Keywords: Cyber-Physical Systems · Mathematical modeling.

1 Introduction

A cyber-physical system (CPS) has been defined as a device with both sensing and reasoning capacities [22]. Strategic initiatives regarding CPSs include "Industrie 4.0" in Germany, the "Industrial Internet of Things (IIoT)" in the United States, and "Society 5.0" in Japan [38]. CPSs typically focus on multidisciplinary applications in healthcare, agriculture, food supply, manufacturing, energy, critical infrastructures, transportation, logistics, security, education [2].

Our interest is in models for driving CPSs, where by "model" we mean a mathematical description of a world aspect [7]. A model describes a law, useful to the extent it generalizes accurately. The development of a model, namely modeling, is close to an art since a model needs to be both "detailed enough", to

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accurately describe phenomena of interest, and “simple enough”, to be amenable to rigorous analysis. Models are applied in a world representation domain.

Classic modeling often regards physical phenomena. In particular, principles /laws of physics, biology and other may be described by parametric algebraic expressions that quantify a functional relation between real variables of interest. Classic modeling ultimately rests upon the conventional measurement process, which is carried out by comparing successively an unknown quantity to a known prototype. For instance, an unknown length is measured by comparing it successively to a known prototype (e.g. a meter) as well as to subdivisions of it. The quotient and remainder of a measurement jointly define a real number – that is how the set R of real numbers emerges. In conclusion, the physical-world is represented by real numbers stemming from measurements, e.g. from inertial sensors, gyroscopes, chronometers, thermometers, microphones, cameras, and other. Therefore, classic models are developed in the Euclidean space R^N, for an integer number N.

However, when humans are involved, then, in addition to (multimodal) sensory data during their interaction with one another, humans also employ cognitive data such as: spoken language, relations, rules, moral principles, concepts and symbols. Therefore, for a seamless interaction with humans, CPSs are expected to be able to cope with cognitive data. In other words, in addition to numerical data stemming from (objective) “physical-world” measurements, a CPS should be able to also deal with non-numerical data stemming from a (subjective) “world-of-mind”. There follows a need to consider a “blended world” model including both the physical-world and a (at least one) world-of-mind.

In response to the aforementioned considerations follows our Proposal P0 including two parts: first, sensory data are numerical, whereas cognitive data may be non-numerical and, second, numerical data and non-numerical data are unified in the context of mathematical lattice theory.

We remark that the emphasis below is mainly on the author’s own publications. For detailed comparisons with specific works from the literature the interested reader might consult the references cited below.

2 The Lattice Computing (LC) Paradigm

Perhaps the most popular approach for dealing with non-numerical data is by ad-hoc transforming them to numerical ones, thus risking the introduction of irreversible and possibly harmful data distortions because original data semantics may be lost. An alternative approach has been proposed based on the fact that popular types of data are partially (lattice) ordered; in conclusion, the data-unifying Lattice Computing (LC) information processing paradigm has been proposed, in principle [7, 10].

Lattice Computing (LC) has been defined as “an evolving collection of tools and methodologies that process lattice ordered data including logic values, numbers, sets, symbols, graphs, etc.,” [18, 39]. We point out that LC is not merely an algorithm but rather it is an information processing paradigm. LC models are expected to be useful in CPS applications including human-robot interaction because LC models can (1) fuse formally numerical data (regarding physical system components) and non-numerical data (regarding cyber system components), (2) compute with semantics, represented by hierarchical partial-order relations, (3) rigorously deal with ambiguity represented by partially-ordered information granules, (4) naturally engage logic and reasoning, (5) process data fast, and (6) deal with both “missing” and “don’t care” data values in a complete lattice [7].

The origins of LC are traced to an application of the fuzzy Adaptive Resonance Theory neural network, or fuzzy-ARTMAP for short, in health care databases towards medical diagnosis [5]; in particular, the fuzzy-ARTMAP operates by conditionally augmenting hyperboxes in the unit hypercube. It was realized that the set of hyperboxes is lattice-ordered; hence, improvements were sought using lattice theory. A naive theory of perception was proposed in the unit hypercube by introducing novel tools such as an “inclusion measure” function for computing a fuzzy degree of inclusion of a hyperbox into another one; moreover, the notion “fuzzy lattice” was introduced in R^N. Nevertheless, the work in [5] was primarily oriented toward medical diagnosis rather than toward theoretical substantiation.

Subsequent work has extended the applicability domain from R^N to a Cartesian product lattice $L = L_1 \times ... \times L_N$ involving disparate, complete lattices. A series of fuzzy lattice neurocomputing (FLN) models was launched and effective applications were demonstrated in pattern recognition [19, 20]. Next, while retaining the basic tools of a FLN model, interest shifted to machine learning [23, 36]. A breakthrough analysis of fuzzy numbers using lattice-ordered “generalized intervals” further turned interest to fuzzy inference systems [11–13]. Later work has introduced a granular extension of Kohonen’s Self-Organizing Map to linguistic data [16]. Moreover, fuzzy lattice reasoning (FLR) was introduced [24] and further employed in a number of applications [13, 15, 17, 18, 20, 26, 29].

Currently, there is a global interest in lattice theory applications in different domains including (Fuzzy) Logic and Reasoning, Mathematical Morphology, Formal Concept Analysis, Computational Intelligence, as outlined next.

Lattice theory has been instrumental in logic [1]. Furthermore, in the introduction of fuzzy set theory, it was pointed out that “fuzzy sets (over a universe of discourse) constitute a distributive lattice with 0 and 1” [43]. Moreover, it was shown how a L(lattice)-fuzzy set generalizes the notion of a fuzzy set [4, 42].

Lattices are popular in mathematical morphology (MM) especially regarding image processing applications [40].

Formal concept analysis (FCA), that is a lattice theory-based field of applied mathematics [3], is based on complete lattice analysis. In the context of FCA several schemes have been proposed for knowledge acquisition, classification, and information retrieval in databases.

Computational Intelligence includes neural computing, which is typically carried out in the Euclidean space R^N. However, there is no evidence that biological neurons operate in R^N. Rather, there is evidence that biological neurons carry out lattice- meet (min) and join (max) operations. Hence, lattice algebra was employed for modeling biological neurons [37]. Different authors have pursued neural computing in the framework of fuzzy lattices [20, 34, 35], where a fuzzy lattice stems from a conventional one by fuzzifying the crisp partial order relation. The latter techniques were extended to fuzzy inference system (FIS) analysis and design [12, 13].

Compared to the employment of mathematical lattice theory in either logic /reasoning or MM or FCA, additional features in LC include (1) complete and/or non-complete lattices, (2) lattices of either finite- or infinite- cardinality, and (3) rigorous mathematical instruments including metric distances as well as fuzzy order functions, based on positive valuation functions for tuning performance. Moreover, LC techniques emphasize data unification based on the fact that popular mathematical lattices include: the Cartesian product R^N, hyperboxes in R^N, Boolean algebras, measure spaces including probability spaces, distribution functions, decision trees, and other.

Synergies/cross-fertilization in LC has been pursued [8–10, 14, 21, 25, 30].

3 Intervals' Numbers (INs)

Currently, the far most popular LC models involve Intervals' Numbers (INs). Recall that an IN is a mathematical object that can represent either a fuzzy interval or a distribution of samples [27, 28, 33]. An advantage of an IN is its capacity to represent data statistics of all-orders using only few numbers; more specifically, L numbers are used to define L intervals, where, typically L=32; hence, a significant data reduction may result in a capacity to process big data fast. No feature extraction is necessary since the all-order statistics, represented by an IN, are implicitly employed as features. Lately, IN-based k nearest neighbor classifiers have been introduced [31, 41].

Applications of INs have been reported regarding neural networks, fuzzy inference systems as well as machine learning [6, 13, 15, 17, 18, 26, 32].

4 Conclusion

The LC paradigm has been proposed for modeling in CPS applications based on a rigorous unification of disparate types of data. New instruments have been introduced in a mathematical lattice such as metric distances as well as fuzzy order functions, based on positive valuation functions. Potential future applications regard effective representations of abstract notions such as "(human) intention" as well as associations of symbols with brain activity patterns toward improving CPSs in practice.

References

1. Birkhoff, G.: Lattice Theory. American Mathematical Society, Colloquium Publications **25**, Providence, RI (1967)
2. CybSPEED: Cyber-Physical Systems for PEdagogical Rehabilitation in Special EDucation. Horizon 2020 MSCA-RISE Project no. 777720, 1 Dec. 2017 - 30 Nov. 2021
3. Ganter, B., Wille, R.: Formal Concept Analysis. Springer, Heidelberg, Germany (1999)
4. Goguen, J. A.: L-fuzzy sets. J Math Analysis and Applications **18**, 145–174 (1967)
5. Kaburlasos, V. G.: Adaptive Resonance Theory with Supervised Learning and Large Database Applications. A University of Nevada, Reno, USA, Ph.D. Dissertation. University Microfilms Inc., US Library of Congress-Copyright Office (1992)
6. Kaburlasos, V. G.: FINs: lattice theoretic tools for improving prediction of sugar production from populations of measurements. IEEE Transactions on Systems, Man and Cybernetics -– Part B **34**(2), 1017–1030 (2004)
7. Kaburlasos, V. G.: Towards a Unified Modeling and Knowledge-Representation Based on Lattice Theory – Computational Intelligence and Soft Computing Applications. Springer, Heidelberg, Germany, series: Studies in Computational Intelligence **27** (2006)
8. Kaburlasos, V. G.: Unified analysis and design of ART/SOM neural networks and fuzzy inference systems based on lattice theory. In: Sandoval, F., Prieto, A., Cabestany, J., Graña, M. (eds.) Computational and Ambient Intelligence, LNCS, vol. **4507**, pp. 80–93. Springer, Heidelberg (2007)
9. Kaburlasos, V. G.: Neural/fuzzy computing based on lattice theory. In: Rabuñal Dopico, J. R., de la Calle, J. D., Pazos Sierra, A. (eds.) Encyclopedia of Artificial Intelligence, pp. 1238–1243. Information Science Reference, IGI Global publication (2009)
10. Kaburlasos, V. G. (ed.): Special Issue on: Information Engineering Applications Based on Lattices. Information Sciences **181**(10), 1771–2060 (2011)
11. Kaburlasos, V. G., Kehagias, A.: Novel fuzzy inference system (FIS) analysis and design based on lattice theory. part I: working principles. International Journal of General Systems **35**(1), 45–67 (2006)
12. Kaburlasos, V. G., Kehagias, A.: Novel fuzzy inference system (FIS) analysis and design based on lattice theory. IEEE Transactions on Fuzzy Systems **15**(2), 243–260 (2007)
13. Kaburlasos, V. G., Kehagias, A.: Fuzzy inference system (FIS) extensions based on the lattice theory. IEEE Transactions on Fuzzy Systems **22**(3), 531–546 (2014)
14. Kaburlasos, V. G., Moussiades, L.: Induction of formal concepts by lattice computing techniques for tunable classification. J. of Engineering Science and Technology Review **7**(1), 1–8 (2014)
15. Kaburlasos, V. G., Pachidis, T.: A Lattice-Computing ensemble for reasoning based on formal fusion of disparate data types, and an industrial dispensing application. Inf. Fusion **16**, 68–83 (2014)
16. Kaburlasos, V. G., Papadakis, S. E.: Granular self-organizing map (grSOM) for structure identification. Neural Networks **19**(5), 623–643 (2006)
17. Kaburlasos, V. G., Papadakis, S. E.: A granular extension of the fuzzy-ARTMAP (FAM) neural classifier based on fuzzy lattice reasoning (FLR). Neurocomputing **72**, 2067–2078 (2009)
18. Kaburlasos, V. G., Papakostas, G. A.: Learning distributions of image features by interactive fuzzy lattice reasoning (FLR) in pattern recognition applications. IEEE Computational Intelligence Magazine **10**(3), 42–51 (2015)

19. Kaburlasos, V. G., Petridis, V.: Fuzzy lattice neurocomputing (FLN): a novel connectionist scheme for versatile learning and decision making by clustering. International Journal of Computers and Their Applications **4**(3), 31–43 (1997)
20. Kaburlasos, V. G., Petridis, V.: Fuzzy lattice neurocomputing (FLN) models. Neural Networks **13**(10), 1145–1170 (2000)
21. Kaburlasos, V. G., Ritter, G. X. (eds.): Computational Intelligence Based on Lattice Theory. Springer, Heidelberg, Germany, series: Studies in Computational Intelligence **67**, 2007
22. Kaburlasos, V., Vrochidou, E.: Social robots for pedagogical rehabilitation: trends and novel modeling principles. In: Dimitrova, M., Wagatsuma, H. (eds.) Cyber-Physical Systems for Social Applications, pp. 1–21. IGI Global, Pennsylvania, USA (2019)
23. Kaburlasos, V. G., Petridis, V., Brett, P., Baker, D.: Estimation of the stapes-bone thickness in stapedotomy surgical procedure using a machine-learning technique. IEEE Transactions on Information Technology in Biomedicine **3**(4), 268–277 (1999)
24. Kaburlasos, V. G., Athanasiadis, I. N., Mitkas, P. A.: Fuzzy lattice reasoning (FLR) classifier and its application for ambient ozone estimation. Intl J. of Approximate Reasoning **45**(1), 152–188 (2007)
25. Kaburlasos, V., Priss, U., Graña, M. (eds.): Proceedings of the Lattice-Based Modeling Workshop, in conjunction with The Sixth Intl Conf. on Concept Lattices and Their Applications (CLA). Palacký University, Olomouc, Czech Republic (2008)
26. Kaburlasos, V. G., Papadakis, S. E., Papakostas, G. A.: Lattice computing extension of the FAM neural classifier for human facial expression recognition. IEEE Transactions on Neural Networks and Learning Systems **24**(10), 1526–1538 (2013)
27. Kaburlasos, V. G., Papakostas, G.A., Pachidis, T., Athinellis, A.: Intervals' numbers (INs) interpolation/extrapolation. In: Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), pp. 1–8 (2013)
28. Kaburlasos, V. G., Vrochidou, E., Panagiotopoulos, F., Aitsidis, C., Jaki, A.: Time series classification in cyber-physical system applications by intervals' numbers techniques. In: Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), pp. 1–6 (2019)
29. Kehagias, A., Petridis, V., Kaburlasos, V. G., Fragkou, P.: A comparison of word- and sense-based text categorization using several classification algorithms. Journal of Intelligent Information Systems **21**(3), 227–247 (2003)
30. Liu, Y., Kaburlasos, V. G., Hatzimichailidis, A. G., Xu, Y.: Toward a synergy of a lattice implication algebra with fuzzy lattice reasoning – a lattice computing approach. In: Papakostas, G.A., Hatzimichailidis, A.G., Kaburlasos, V.G. (eds.) Handbook of Fuzzy Sets Comparison – Theory, Algorithms and Applications. Science Gate Publishing (SGP), vol. 6, pp. 23–42 (2016)
31. Lytridis, C., Lekova, A., Bazinas, C., Manios, M., Kaburlasos, V. G.: WINkNN: Windowed Intervals' Number kNN classifier for efficient time-series applications. Mathematics **8**(3), 413 (2020)
32. Meng, X., Liu, M., Zhou, H., Wu, J., Xu, F., Wu, Q.: Fuzzy c-means on metric lattice. Automatic Control and Computer Sciences **54**(1), 30–38 (2020)
33. Papadakis, S. E., Kaburlasos, V. G.: Piecewise-linear approximation of nonlinear models based on probabilistically/possibilistically interpreted Intervals' Numbers (INs). Information Sciences **180**(24), 5060–5076 (2010)
34. Petridis, V., Kaburlasos, V. G.: Fuzzy lattice neural network (FLNN): A hybrid model for learning. IEEE Trans Neural Networks **9**(5), 877–890 (1998)

35. Petridis, V., Kaburlasos, V. G.: Clustering and classification in structured data domains using Fuzzy Lattice Neurocomputing (FLN). IEEE Trans Knowledge Data Engineering **13**(2), 245–260 (2001)
36. Petridis, V., Kaburlasos, V. G.: FINkNN: a fuzzy interval number k-nearest neighbor classifier for prediction of sugar production from populations of samples. J. Machine Learning Research **4**, 17–37 (2003)
37. Ritter, G. X., Urcid, G.: Lattice algebra approach to single-neuron computation. IEEE Trans Neural Networks **14**(2), 282–295 (2003)
38. Serpanos, D.: The cyber-physical systems revolution. Computer **51**(3), 70–73 (2018)
39. Sussner, P., Campiotti, I.: Extreme learning machine for a new hybrid morphological/linear perceptron. Neural Networks **123**, 288–298 (2020)
40. Sussner, P., Ritter, G. X.: Decomposition of gray-scale morphological templates using the rank method. IEEE Trans Pattern Analysis and Machine Intelligence **19**(6), 649–658 (1997)
41. Vrochidou, E., Lytridis, C., Bazinas, C., Papakostas, G. A., Kaburlasos, V. G.: Fuzzy lattice reasoning for brain signal classification. (under review)
42. Xu, Y., Ruan, D., Qin, K., Liu, J.: Lattice-Valued Logic. Springer, Berlin, Germany (2003)
43. Zadeh, L. A.: Fuzzy sets. Inform Contr. **8**, 338–353 (1965)