

Formal Concept Analysis from the Standpoint of Possibility Theory

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Abstract. Formal concept analysis (FCA) and possibility theory (PoTh) are two theoretical frameworks that are addressing different concerns in the processing of information. Namely FCA builds concepts from a relation linking objects to the properties they satisfy, which has applications in data mining, clustering and related fields, while PoTh deals with the modeling of (graded) epistemic uncertainty. This difference of focus explains why the two settings have been developed completely independently for a very long time. However, it is possible to build a formal analogy between FCA and PoTh. Both theories heavily rely on the comparison of sets, in terms of containment or overlap. The four set-functions at work in PoTh actually determine all possible relative positions of two sets. Then the FCA operator defining the set of objects sharing a set of properties, which is at the basis of the definition of formal concepts, appears to be the counterpart of the set function expressing strong (or guaranteed) possibility in PoTh. Then, it suggests that the three other set functions existing in PoTh should also make sense in FCA, which leads to consider their FCA counterparts and new fixed point equations in terms of the new operators. One of these pairs of equations, paralleling the one defining formal concepts, define independent sub-contexts of objects and properties that have nothing in common.

The parallel of FCA with PoTh can still be made more striking using a cube of opposition (a device extending the traditional square of opposition existing in logic, and exhibiting a structure at work in many theories aiming at representing some aspects of the handling of information). The parallel of FCA with PoTh extends to conceptual pattern structures, where objects, may, e.g., be described by possibilistic knowledge bases.

In the talk we shall indicate various issues pertaining to FCA that could be worth studying in the future. For instance, the object-property links in formal contexts of FCA may be a matter of degree. These degrees may refer to very different notions, such as the degree of satisfaction of a gradual property, the degree of certainty that an object has, or not, a property, or still the typicality of an object with respect to a set of properties. These different intended semantics call for distinct manners of handling the degrees, as advocated in the presentation. Lastly, applications of FCA to the mining of association rules, to the fusion of conflicting pieces of information issued from multiple sources, to clustering of sets of objects on the basis of approximate concepts, or to the building of conceptual analogical proportions, will be discussed as other examples of lines of interest for further research.