Some applications of Lattice Analysis (1983-2013)

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Abstract. Following [1,2] we report on applications of Lattice Analysis either for deciphering data or for clarifying abstract lattices. Here, lattices are often considered as implication models that can be summarized with canonical basis [3,1,4,5] or (semi) lattice cores [1]. In a more symmetric way decompositions through lattice congruence / tolerance relations are used for real data analysis as well as for getting understandable structures of abstract lattices [6,7 and below]. As for the needed algorithms, many efforts have been done to "overtake" the NEXT-CLOSURE algorithms since their discovery in 1984 [5]. For implications the fees may involve an exponential explosion in memory. We will just try to give some visions of what could be next in doing with(-out) NEXT-CLOSURE. Hence in a fresh original spirit of the early eighties, for all these and further developments we still promote "more simplicity with more structure" (and tolerances ...) for deepening the concept systems and lattice applications.

Keywords: closure operator, lattice, canonical basis of implications, quasi / pseudo-closed, (semi) lattice cores, perspectivities / arrows, congruences / tolerances, combinatorial exhaustive enumeration, NEXT-CLOSURE algorithms.

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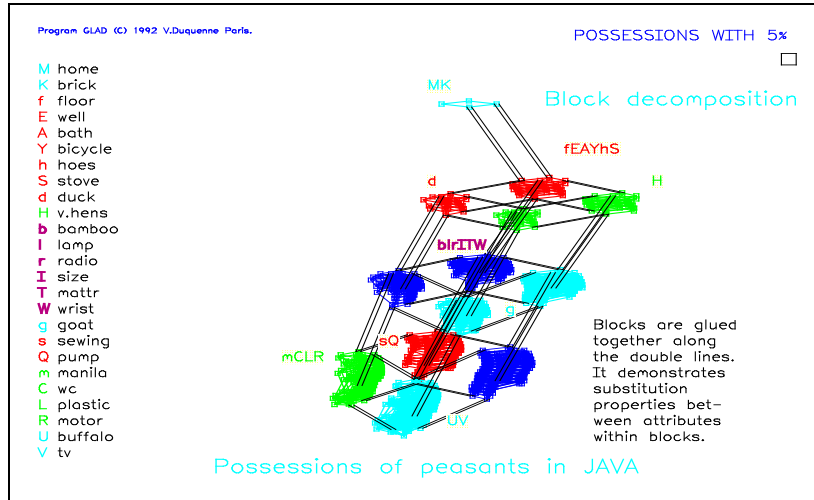
Fig. 1. Peasants x possessions. Lattice gluing decomposable, hence substitution properties...
From : Models of possessions and Lattice Analysis, *Social Sci. Information* (1995).
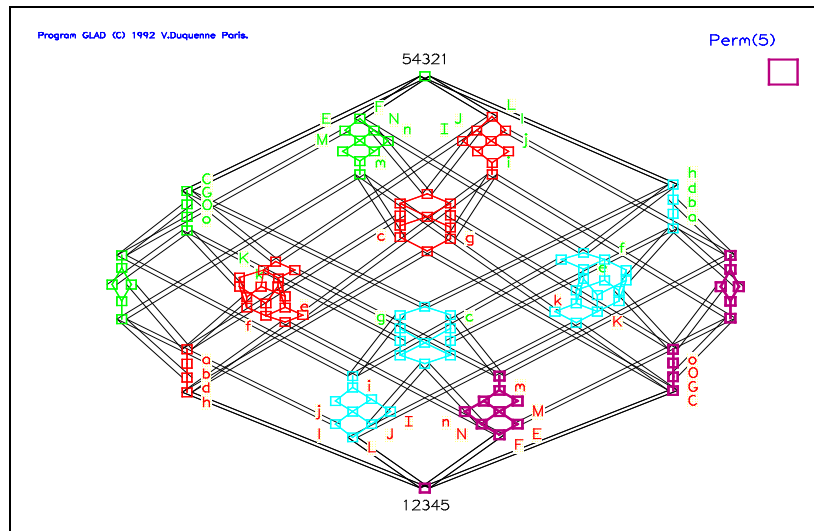
Fig. 2. Perm(5) quotiented by the meet of its maximal congruences: "having the same comple-
ments". From : On permutation lattices, *Mathematical Social Sciences* (1994).