Projective Lattices

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A lattice $\mathbf{L}$ is *projective* in a variety $\mathcal{V}$ of lattices if whenever

$$f : \mathbf{K} \twoheadrightarrow \mathbf{L} \tag{1}$$

is an epimorphism, there is a homomorphism

$$g : \mathbf{L} \to \mathbf{K} \tag{2}$$

such that $f(g(a)) = a$ for all $a \in L$.

Projective lattices are characterized in [3] by four conditions. This talk will discuss two of them that are of current interest.

If g in (2) is only required to be order-preserving, it is called an *isotone section* of the epimorphism (1). We will characterize which lattices $\mathbf{L}$ have an isotope section for every epimorphism (1). We will use this to characterize when the ordinal (linear) sum of two projective lattices in $\mathcal{V}$ will be projective and give some surprising examples.

The second of the four conditions characterizing projectivity we will discuss is join refinement and the dependency relation; the so-called D-relation. This condition and some closely related concepts are used in many parts of lattice theory. Besides free lattice, projective lattices and finitely presented lattices, it has applications to transferable lattices, congruence lattices of lattices, representing finite lattices as congruence lattices of finite algebras, and ordered direct bases in database theory [1, 2].

References

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