

Lifeworld and Mathematics

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Abstract. The article “Lifeworld and Mathematics” has been inspired by well-known scientists, from whom are listed here: Edmund Husserl, Jürgen Habermas, Reuben Hersh, Martin Heidegger, Hartmut von Hentig and Knut Radbruch. Basic for this article is Husserl’s phenomenological *lifeworld analysis* of the mathematized modern natural science. Habermas, in whose social theory the concept of lifeworld has also a central meaning, recommends a theoretic communicational approach for the lifeworld analysis which answers the question about the intersubjective constitution of the lifeworld in the sense of the american pragmatism. Hersh has a pragmatic understanding of mathematics so that he views mathematics as a social, cultural, historical reality. Heidegger saw the basic character of the modern knowledge attitude in the new knowledge claim named the “mathematical” which is not deducable out of mathematics. Hentig has offered the book “Magier oder Magister? Über die Einheit der Wissenschaft im Verständigungsprozess”, in which he discusses generally the role of the science in our society, while doing so he considers as necessary a democratically activated general understanding of scientific actions. Already 1991 Radbruch has pointed out in a lecture at the Darmstadt Seminar about “general mathematics” the analysis of conditions and realizations of the mathematical as task of general mathematics.

1 Lifeworld and Inworld

The concept “*lifeworld*” has been introduced into philosophy by Edmund Husserl in his late work “*The crisis of the european sciences and the transcendental phenomenology*” (see [Hu54]). With the term “lifeworld” Husserl names - in the frame of his phenomenological philosophy - the concrete context of the world. This context is intersubjectively experienced by humans in original evidence and is therefore ordered before the objective-scientific cognition of the world. Husserl founded on the “lifeworld” concept his fundamental critic about the development of science. For him the scientists have chanced the ideal of objectivity, which has to be understood only methodically, into an independently made “objectivism” by which the relationship of the research to the responsibility of the acting human being is lost. To overcome by it the occuring sense-crisis of science, Husserl recommends to make conscious that and how the objectivistic imagined world

arises out of human achievements which are founded on the lifeworld as the extensive horizon of human cognition and action.

Husserl's phenomenological *lifeworld analysis* of the mathematized modern natural science has been extended by Alfred Schütz with a phenomenological analysis of the social lifeworld [SL79]. Schütz understands the lifeworld as the transcendental frame of possible everyday experiences of the recognizing and acting subject. Jürgen Habermas, in whose social theory the concept of lifeworld has also a central meaning, criticizes the approach of Schütz: On the one hand Schütz tends to start out from an intersubjectively constituted lifeworld, without to clarify how the lifeworld is intersubjectively produced (a problem of which Schütz knew that also Husserl has not solved it); on the other hand he takes over Husserl's philosophical view of consciousness to comprehend the "experiencing subject" as the last reference point of the lifeworld analysis [Ha81; vol.2, p.197f]. Habermas recommends a theoretic communicational approach for the lifeworld analysis which answers the question about the intersubjective constitution of the lifeworld in the sense of the american pragmatism. A convincing understanding of mathematics has also to be pragmatically founded, as it is for instance realized by Reuben Hersh in his 1997 appeared book "What is mathematics, really?" [He97]. Hence, the treatment of the theme "lifeworld and mathematics" shall be based in this paper on Habermas' concept of lifeworld, which has therefore be more explicated in detail.

Jürgen Habermas introduces in his main work "Theorie des kommunikativen Handelns" [Ha81] "lifeworld" as a complementary concept to the "*communicative action*". Habermas understands action and therefore communicative action as mastering of situations. "The concept of communicative action cuts out of the mastering of situations first of all two aspects: the *teleological aspect* of the realization of the mastering of purposes (or the execution of a plan of action) and the *communicative aspect* of the interpretation of a situation and the achieving of an agreement. In the communicative action the participants pursue their plans on the basis of common definitions of situations in mutual agreements." [Ha81; vol.2, p.193] For the unification of common definitions of situations the *lifeworld* is fundamental because "communicatively acting subjects communicate always in the horizon of a lifeworld. [...] This lifeworld background serves as a source for defining situations which are presupposed by the participants as unproblematic." [Ha81, vo.1, p.107] For the situation-oriented communication "the lifeworld is a reservoir of self-evident facts or unshakable convictions which the communication participants utilize for interpretation processes. But single elements and determined self-evident facts are only mobilized in the form of consentaneous and at the same time of problematic knowledge if they become relevant for some situation. [...Out of an understanding-oriented view] we are able to think the lifeworld as a culturally transmitted and linguistically organized supply of explanation patterns." [Ha81; vol.2, p.189]

To be able in answering the question about the intersubjective *constitution and reproduction of the lifeworld*, Habermas goes back to the social psychology of the american pragmatist Georg Herbert Mead. Mead, who is interested

in the complementary structure of the subjective and social world, describes in his book “Mind, Self, and Society” how speech becomes a medium of socialization and social integration and how personal identities and social institutions develop through linguistic arrangements and normative steering interactions. Socialization and social integration, which produce symbolic structures of the self and the society and with it competences and patterns of relationships, are executed by acts of communication; however Mead does not analyse thematically that these processes of communications are also reflected in intersubjectively constituted cultural knowledge. To make it more clear that the speech mediated normative interaction - besides subjective competences and social patterns of relations - also leads to a propositional differentiated linguistic communication, Habermas distinguishes more clearly as Mead “*between speech as a medium of communication and speech as a medium of the coordination of actions and of the incorporation of individuals.*” [Ha81; vol.2, p.41]

This treefold function takes over the speech in the communicative action: “Under the functional *aspect of communication*, the communicative action serves the tradition and renewing of cultural knowledge; under the *aspect of the coordination of action*, it serves the social integration and the production of solidarity; finally, under the *aspect of socialization*, the communicative action serves the instruction of personal identities.” [Ha81; vol.2, p.208] In the communicative action the constitution and reproduction of the lifeworld take place and to be more precise by renewing and continuing of valid knowledge, through development and stabilisation of legitimate orders and of solidarity of groups as well as by the training of sound of mind and competent acting persons. Habermas goes with his lifeworld understanding beyond the area of knowledge because “the solidarities of the about values and norms integrated groups and the competences of incorporated individuals in similar manner as cultural tradition [...] flow in communicative action.” [Ha81; vol.2, p.205]

As a reference system for describing and explaining, which concerns a lifeworld as a whole and do not only regard occurrences, Habermas turned out *culture*, *society*, and *person* as the structural components of the lifeworld which corresponds the proceedings of the *cultural reproduction*, the *social integration*, and the *socialization*. What Habermas wants to understand in this connection by culture, society, and person, he lays down as follows: “I entitle “*culture*” as the knowledge supply out of which the communication partners provide themselves with interpretations to come to an agreement about something in the world. I entitle “*society*” as the legitimate order about which the communication participants organize their membership in social groups and secure with it their solidarity. I understand under “*Personality*” the competences which makes a subject to have speech and capacity to act, i.e. to renovate, to participate in communications and at the same time to declare the own identity.” [Ha81; vol.2, p.209]

To make clear how the lifeworld differentiates itself in its symbolic structures, it recommends to describe in detail the three processes of reproduction and their connections:

- “The *cultural reproduction* of the lifeworld [...] secures the continuity of the tradition and a sufficient coherence of knowledge for the everyday practice, respectively. Continuity and coherence are measured by the rationality of the as valid excepted knowledge.” Disturbances of the cultural reproduction cause that the as valid excepted schemata refuse and “the resource $> \textit{sense} <$ becomes scarce.” [Ha81; vol.2, p.212f.]
- “The *social integration* [...] provides for the coordination of action about legitimately regulated interpersonal relations and makes constant the identity of groups in a sufficient measure for the everyday practice. For this the coordination of action and the stabilization of group identities are measured by the solidarity of the relatives.” Disturbances of the social integration cause that the as legitimately regulated memberships are not sufficient and “the resource $> \textit{social solidarity} <$ becomes scarce.” [Ha81; vol.2, p.213]
- “The *socialization* of the relatives of a lifeworld [...] secures for the again growing generations the acquisition of generalized abilities of action and provides for the voting of individual life stories and collective life forms. Interactive abilities and styles of life guidance are measured at the soundness of mind of the relatives. Disturbances of the socialization course of events produce that persons can protect their respective identity only with defensive strategies which impair a fair realistic participation in interactions through which “the resource $> \textit{self - strength} <$ becomes scarce.” [Ha81; vol.2, p.213]

After it has been described what the processes of reduction are capable to achieve for receiving in each case the belonging component of the lifeworld, the question remains what it can contribute respectively for receiving the two other components. The *cultural reproduction* can stabilize legitimations for existent institutions in the society as well as deliver for the person effectively forming attitude patterns for the acquisition of generalized capacity to act. The *sozial integration* can secure in the culture moral duties and commitments as well as stabilize for the person legitimately regulated social memberships. The *socialization* can promote in the culture interpretation efforts as well as renew in the society motivations for concurring norm actions. (s. [Ha81; vol.2, p.214ff.])

Although Habermas takes over from Mead the processes of socializations and social integration for the structure of the subjective and social world, he criticizes that Mead establishes these processes alone *ontogenetically* and does not clarify particularly the transition to norm conducted interaction as well as the structure of group identities. To close this gap, Habermas refers to the 1912 written sociology of religions of Emil Durkheim [Du81] in which the roots of the moral authority of social norms are uncovered. Durkheim analysed the religious belief and the patriotism “as expression of a tribe development deeply rooted in a collective consciousness which is constitutive for the identity of groups.” [Ha81; vol.2, p.206] Habermas sees in this usage of speech in ritual secured normative consent, which produce this collective consciousness, the *phylogenetic* foundation for the linguistically mediated norm-conducted interaction, which the starting position presents for the social-cultural development.

Where and how can one grasp *empirically* what for Habermas the lifeworld is? Since Habermas speaks usually about the lifeworld of social groups, this suggests to examine the lifeworld structures of those social groups. For this Habermas holds the participant perspective for unsuited and recommends more likely “*the everyday concept of lifeworld* with whose help communicatively acting persons localize and date themselves and their statements in social rooms and historical times. The persons meet themselves in the communicative everyday speech not only in the attitude of participants but they also give narrative descriptions of occurrences which happen in the context of their lifeworld.” [Ha81; vol.2, p.206] This practice of narrations particularly has the function that the communication participants ascertain their personal and collective self-understanding. A *personal identity* can only be trained if the succession of the own actions is understood as a narratively representable life-story, and a *social identity* only if the own story is recognized as interactively embedded into the narratively representable story of collectives. “The collectives obtain their identity only to such an extent as the expectations, which people make themselves about the lifeworld, overlap sufficiently and concentrate to unproblematic background convictions.” [Ha81; vol.2, p.206]

For this it recommends itself to examine the lifeworld of a social group on the basis of the expectation which the group members make themselves about their lifeworld, respectively. However, it has to be clarified that this approach turns around the line of sight: The individual is as a rule member of different social groups of which the examination group is only one; hence, of the connecting lifeworld expectations the single members of the examining group have not only the overlaps to be identified but it has also to recognize the relation to the examination group, respectively. To make these connections more capable of thinking, it offers to introduce the net of all lifeworlds, which are represented in the knowledge, behavior, and self-understanding of *one person*, as theoretical concept of the lifeworld analysis; this concept shall be named the “*inworld*” of this person. As an inversion of the inworld-definition one obtains the statement: the lifeworld of a social group is the common of the inworlds of their members. Obviously this statement reflects the propagated initial search stage.

To found this attempt further for the empirical lifeworld analysis, the inworld concept shall be directly anchored into the social-psychology of Mead as already Habermas has done it with his lifeworld concept. For this it has been returned to Mead’s theory of identity whose structural characteristic is that Mead subdivides the *self-identity* in a subjective]I[and a social]Me[. The]I[is spontaneous, emotional, creative, and expressive, while the]Me[is reflected, rational, conventional, and controlled. The]I[gives me the self-feeling, the]Me[represents the we-experience. Mead writes himself: “The]I[is the reaction of the organism on the attitudes of others [...]. The attitudes of the others form the organized]Me[, and one reacts on it as an]I[. [...] For the human it is important that he receives the attitudes of others and adjusts his own identity or takes on the battle. This recognizing of the identity of the single in the process of the identity consciousness gives him the attitude of the self-assertion or subordination under

the community. Through it he reaches to a definitive identity.” [Me73; p.218, 237] The]I[and the]Me[are therefore constitutive for the identity formation of humans.

The *representations of the own inworld* have obviously their place in the]Me[; there they will be constituted and traditionalized by the processes of the cultural reproduction, the social integration, and the socialization taking place in the social groups of the lifeworlds which belong to the inworld. That and how these processes can be effective, Mead clarifies with the spiritualization of the gesture arranged interaction: “The transmission of gestures is a part of the proceeding social processes. [...] The development of the language, in particular of the significant symbols, made it possible that just this external social situation is included in the attitude of the individual.” [Me73; S.230] According to the theory of Mead, the *cultural reproduction* for the knowledge and thinking of the individual is essential because “one must insert the external social world, which one has entered into one-self to be able to think.” [Me73; S.243] With this the]I[and the]Me[are absolutely necessary. For the *social integration* “one must take over the attitudes of the other to belong to a community; [...] on the other hand the individual reacts constantly on the social attitudes and changes in this process just this community.” [Me73; p.243] Finally Mead makes available also the special meaning of the *socialization*: “We can only realize ourselves in so far as we recognize the other in his connection to us. While the individual takes over the attitude of the other, he is capable to realize himself as identity. [Me73; p.273]

In the sense of Mead one can say: the *inworld of a person* comes into being in that way, that the lifeworld attitudes of the others, which belong to the same social groups as the person, are getting in the]Me[while doing so the respective reference is taken to the social group it belongs to. With the takeover of the lifeworld attitudes the]Me[gives a form to the]I[and with it to the self-identity which is determined conventionally [Me73; p.253]; i.e. the inworld delivers for the person’s thinking and acting a conventional background which is in particular important for the identity formation. Since the lifeworld attitudes of the others are an expression of the collective consciousness of a social group, the inworld concept is understood both *ontogenetic* and also *phylogenetic*. It shall be again clarified: the concept of inworld builds on Habermas’ concept of lifeworld for whose arguments Habermas goes back to Mead’s theory of identity and Dürkheim’s theory of collective consciousness; constitution and reproduction takes place in the inworlds - as in the lifeworlds - in the communication of action.

2 Mathematics and the Mathematical

What is now the theme of mathematics in this paper? About which understanding of mathematics can we use for our theme? This is not easy to answer because it has always been difficult to say what mathematics is all about. Even the *commonly given definitions of mathematics*, as they are presented in the current books of reference, are scarcely sufficient. Deputizing the 1998 edition of the

Brockhaus-Enzyklopädie [Br98] we obtain:

“*Mathematics* [is] one of the oldest sciences, emerged out of tasks such as counting, calculating and measuring, which were based on practical (first of all scientific and technical) formulations of questions for whose treatments originally numbers and geometric figures as well as their mutual connections were taken. Thereby the concept of number and the elementary geometric concepts have been developed. Until today mathematics receives strong impulses from the attempt to contribute to the description of scientific, economic etc. occurrences. The field of activity of mathematics was essentially extended by abstracting the original meaning of the studied objects and led to a “science of formal systems” (D.Hilbert). According to that one understands under *modern mathematics* the science of the abstract structures and logical conclusions, which are determined by commitments of a few basic acceptances about the relations and connections between elements of arbitrary size. It belongs to its essential tasks to set up the most general connections without contradictions between these quantities, out of which it results conclusions in form of statements (theorems) on a purely logical path. Mathematics is characterized by a high precision of its concept system, strictness of its methods of proving and a strongly deductive character of its presentation.”

The cited text describes obviously the common understanding of mathematics which dominates under mathematicians and does this definitely by a concise and convincing manner. So what is unsatisfactory about this concrete definition of mathematics? To say it shortly: it lies on that what is left - in general the relation of mathematics to the community of communication of thinking and acting humans. Fortunately, in 1997 there has been appeared a book which presents and substantiates a human related pragmatic view of mathematics: it is the already mentioned book “*What is mathematics, really?*” [He97] of the mathematician and philosopher Reuben Hersh. The approach of Hersh is based on that what mathematicians really need and do for their work, and is not based on a somehow ideal of mathematical objects and activities. Typical for Hersh is the following course: The mathematical work comes thereby into action that one discovers a problem which is connected with the existing mathematical culture; then one works on a problem and needs help and encouragement in the case of present difficulties; one suggests finally a solution for which one needs agreement and critics. How isolated and self-absorbtionally a mathematician may work, he finds nevertheless organ and confirmation of his work in the community of mathematicians. [He97; p.5]

His pragmatic understanding of mathematics delivers Hersh also the arguments to keep one’s distance to the formalism and the platonism, the two main opinions of the nature of mathematics. [He97; p.7ff] Concerning the *formalism* he criticizes that mathematics may be understood in parts as a rule guided game, but with that it remains unclear how rules can be made, developed and evaluated with regard to their applications. He reproaches the *platonism* that it does not give answers how the immaterial mathematical objects may come in contact

with mathematicians out of flesh and blood and how the strange parallel existence of the physical and the mathematical reality can be explained. For Hersh these questions can be only answered when one understands mathematics as a *social, cultural, historical reality*. In this sense he describes the mathematics as follows:

“A world of ideas exists, created by human beings, existing in their shared consciousness. These ideas have objective properties, in the same sense that material objects have objective properties. The construction of proof and counterexample is the method of discovering the properties of these ideas. This branch of knowledge is called mathematics.” [He97; p.19]

In the second part of his book, Hersh refers views about mathematics out of the philosophical history from Pythagoras until the present time; for this he describes two parallel running developments: that of the “*main stream*” at which mathematics is respected as superhuman, abstract, ideal, unending, and eternal, and that of the “*humanists and mavericks*” at which mathematics is understood as a human activity and as human creation [He97; p.91f.]. Hersh counts to the main stream first of all Pythagoras, Plato, Descartes, Spinoza, Leibniz, Kant, Husserl, Frege, Russell, Carnap, to the humanists and mavericks Aristotle, Euclide, Locke, Hume, Mill, Poincaré, Sellars, Wittgenstein, Popper, Lakatos, Wang, Tymoczko, and Kitcher. It is surprising that Hersh reckoned Husserl to the main stream and nevertheless quoted comprehensively out of Husserl’s late work as for instance: “without the ‘what’ and the ‘how’ of its pre-scientific materials, geometry would be a tradition empty of meaning.” [He97; p.166] The lifeworld conception, which is here mentioned by ‘pre-scientific materials’ and honours Husserl more as a “humanist”, does not come up without a reason in connection with geometry because the geometry is the science from which the concept “lifeworld” is introduced into Husserl’s late work [Hu54].

This first explicit connection of *mathematics and lifeworld* shall be shortly presented in its basic thoughts. First Husserl explains the change of the antique conception of science to the modern science conception. For him the unheard new finding was the “idea of a rational infinite universe with a systematic rationally controlled science”. He clarified this idea by rearranging the euclidean geometry to the modern geometry: The finally closed apriori of the greek geometry changes to a universal apriori to which the infinite ideal space belongs and an “infinitely - despite the infinity - in itself closed homogeneous systematic theory, which allows to construct - from axiomatic concepts and theory ascending - each thinkable drawn shape in the space of deductive uniqueness. [...] Interested for these ideal shapes and consequently concerned with them, to determine them and to construct out of already determined shapes new ones, we are ‘Geometers. And just as equally, also for the further sphere, which is also concerned with the dimension of the number, we are mathematicians of the ‘pure shapes, the universal form of which is the self idealized space-time-form. [...] How all through human contributions arising achievements they [the pure limes shapes] remain objectively recognizable and available, also without that their sense formings must be explicitly renewed.

But now it happened in the course of history that the theory of the objectively intended ideal forms developed itself into a systematic, thoroughly useful science - the *modern mathematics* - and made itself independent in the view of autonomous objectivity, without that a real understanding of the actual sense and the internal necessity of the constituted achievement of abstraction was present. According to Husserl “the real evidence is missing and missing further, in which the perceiving-achieving human can give himself an account not only about that, what he is doing now and with which he is operating, but also about all through sediment resp. tradition closed sense-implications, hence about the permanent assumptions of his creations, concepts, statements, and theories. Is the science and its method not equal to an obviously very useful and reliable machine which everybody can learn to handle correctly, without to understand at least the internal possibility and necessity of such services?” [Hu54; p.52] To be able to counter the bad state of affair, Husserl retains it for urgent to reflect the relation to the lifeworld - “the in our concrete worldlife us permanently as real given world” - and to the human as its subject.

From the philosophical view this means according to Husserl : “Clearness about the origin of the modern intellect and with it - capable of the not highly enough evaluated importance of mathematics and the exact natural sciences - about the origin of these sciences [... since the exact natural science] from the beginning and furthermore in all its changing of senses and irrelevant sense interpretations of decided meaning for growing and being of the modern positive sciences, in the same manner of the modern philosophy - really of the spirit of the modern european humanity in the first place .” [Hu54; p.58f]

More detailed as Husserl, his follower Martin Heidegger has treated the question about the *basic understanding of the modern natural science* and - to be more precise - in his 1935/36 presented lecture “Grundfragen der neuzeitlichen Metaphysik”, of which the text has been published 1962 as a book with the title “Die Frage nach dem Ding” [Hd62]. Heidegger characterizes the basic feature of the modern natural science as follows: “The basic feature has to consist of that, what the basic movement of the science as such standardly dominates as equally original: it is the work intercourse with the things and the metaphysical model of the thingness of the things.” [Hd62; p.52] Heidegger sees this basic character of the modern knowledge attitude in the new knowledge claim which he names the “mathematical”.

The “*mathematical*” is according to Heidegger not deducible out of mathematics, but mathematics is itself only a determined form of the mathematical (cf. [Ra91], [Wi98]). For being able to determine what is meant with “the mathematical”, Heidegger goes back to the origin of the word in old greek: *τα μαθηματα* means the learnable. For Heidegger the learning is at this a taking and an acquisition, by which the taking occurs through knowledge-taking and the acquisition through application. Since Heidegger views human knowledge in the basic position of thinking to the things, respectively, and to the being anyhow, it is for him “the basic learning of such taking where we this, what at all a thing is, take into the knowledge. [...] This original learning is [...] some learning [...], at which

the receiver takes only that, what he already has for some reason.” [Hd62; p.56] From that it results for Heidegger: “The *ματεματα*, the mathematical, that is such at the things what we actually already know, what we therefore do not take out of the things, but already in a certain way bringing them with us.” [Hd62; p.57] Differently said: “The mathematical is such basic position to the things, in which we make us clear what they are already presented us. The mathematical is therefore the basic assumption of the knowledge of the things.” [Hd62; p.58] Herewith Heidegger sees the central significance of the mathematical for modern thinking, since in the nature of the mathematical there lies a “will for the new design and self-foundation of the knowledge form as such.” [Hd62; p.75]

As what can the mathematical grasp thinking and acting? In thinking the mathematical is the actual learnable about the things and delivers with it the knowledge form, i.e. it is a formal thinking or more pointed: a *form thinking*. For Heidegger the nature of the mathematical form thinking lies in its marking as *sketch* by which it is settled “what we actually hold about the things, as what they and how they are recognized in advance.” [Hd62; p.71] Such an axiomatic sketch delimits the area of the things which are thought by the sketch in the given knowledge form. In the mathematical as such sketch thinking, Heidegger sees the basic feature of modern thinking and knowledge, with which it is meant in particular a liberation from the middle-age thinking to the reformation and self-foundation of the knowledge form. Since in his book [Hd62] about the confrontation with Kant’s “critics of pure reason” Heidegger focusses the determination of the mathematical on the taking in learning and moves back the acquiring with the argument that the taking (as knowlegde) precedes necessarily the using (as appropriation). [Hd62; p.55]

If one however changes the relation of Kant’s transcendental logic into Peirce’s pragmatism, as Karl-Otto Apel has presented it in his article “Von Kant zu Peirce” [Ap76], then the objective validity of knowledge for the single consciousness stands not more in the foreground, but the intersubjective communication about truth, rightness, and truthfulness of statements; i.e. the claim of objective rationality will be replaced by the communicative rationality. The *pragmatic reference* allows to see the taking of knowledge and the using of appropriation in their interrelation, because each using produces also something more of taking. Pragmatically the mathematical has to be understood as form thinking which causes a mutual process of conception and application. In this way the mathematical thinking is always constituted intersubjectively, because the figures and operations of thinking have already been formed by processes in a community of communication. The mathematical has therefore its place in the lifeworld of the scientifically oriented communities of communication where “lifeworld” is understood here in the sense of Habermas.

But in which relationship does mathematics stand to this lifeworld? For clarifying this, it must be made understandable how it comes to the specific forms of the mathematical which we call “mathematics”. According to Hersh’s understanding of mathematics, there are recognizing, thinking, and communicating humans which cause these formings. Out of figures and operations of thinking

concerning their mathematical forms, which are always activated again and again in communications, there are formed in a process of progressive conventionalizations determined formal systems of knowledge, which constitute a culture of formal thinking: the *mathematics*. It is frequently difficult to say why certain formal systems of thinking belong to the respective mathematics-culture and others not; for instance, the system of integral proportions belongs to today's mathematics, but not the tone system of music. What finally a mathematic-culture takes up from the conventionalized form-thinking (or again separates out; see [La72]), that is decided in a complex process in the respective community of communication of mathematicians. There are the integrated conventions of thinking and the connected explanation patterns and convictions which make up the lifeworld of the respective mathematics-culture. With the progressive specialization, the community of communication of mathematicians becomes subdivided into further subgroups which respectively form their own lifeworlds what the general communication about mathematics makes it increasingly more difficult. Thus, it is today not easy to find out what still belongs actually to the lifeworld of the whole community of mathematicians. The real question about the relation of mathematics to the *lifeworld of our present society* escapes therefore a simple answer.

3 Inworld and Mathematics

In view of the considered difficulties, not the relation of mathematics to the lifeworld of our society shall be directly looked for, but the mathematical and the mathematics in the *educational process of the single*, i.e. in the constitution and reproduction of the inworld of social individuals. The extension to the mathematical recommends itself because also in the educational process of an individual the conventional mathematics as a forming of the mathematical is acquired. According to Jean Piaget, the *evolutional psychological roots* lie for the education of the mathematical in the existing coordinates of actions which were already present before the development of speech; then “these coordinates form a logic of actions which, as far as she is concerned, presents the starting point for logic-mathematical structures.” [Pi73; p.51] In Mead's description of the development of speech (from the gesture helps over the symbol helps to the norm regulated interaction) it becomes clear how further form thinking develops. In general the cultural communication, the social integration and the soziation contribute so early for it that a fullness of basic patterns of the mathematical could be acquired. How far these basic patterns are activated during the learning of the mathematical, this has been studied impressively by the *interpretative teaching research* with the key concept of the so-called “framing”. (s. [Go80], [MV91])

Nevertheless it is up to now not much known about what *the mathematical and the mathematics* mean for the constitution and reproduction of the inworld of single humans. How difficult answers are about this question, it refers already on that ambivalent views of many humans about mathematics. The global opinion that arithmetical and logical thinking is learned through teaching mathemat-

ics is in any case no help as long as it cannot be satisfactorily answered what is meant and wanted, respectively, with arithmetical and logical thinking. As a special obstacle there is the dominance of the mechanistical understanding of mathematics as it is for instance expressed in the narrowing of the understanding of logic as the doctrine of the forms of thinking onto calculations of logical conclusions. An even more dangerous development is the progressive specialization and instrumentalization in mathematical research which reduces the coherence and continuity of the common knowledge in the society of mathematicians and produces in this way an always larger distance between mathematics and the world.

For mathematics Husserl's demand is urgent as before to remember the relation to "the in our concrete world-live us as constandy real given world" and to the human as its subject. In the sense of Husserl's demand we tried hard in our Darmstadt research group since more than twenty years to answer the question: What can and shall mathematics be for the general public? [Wi96b] For answering this question we had to clarify the conception of the self-understanding of mathematics, of its relationship to the world as well as the questions about sense, meaning, and connection of mathematical action. We received in particular great help by the book "*Magier oder Magister? Über die Einheit der Wissenschaft im Verständigungsprozess*" [Hn74], in which Hartmut von Hentig discusses generally the role of the science in our society, while doing so he considers as necessary a democratically activated general understanding of scientific actions. The sciences must, as Hentig demands, "examine their disciplinarity and this means

- to uncover their unconscious purposes,
- to declare their conscious purposes,
- to select and to straighten their means,
- to present publicly and understandably their possible consequences and
- to make accessible their way of discovery and results about the common language." [Hn74; p.136f.] "The always more necessary growing restructuring of sciences in themselves - to make them better learnable, mutually available, and more generally criticizable (i.e. also beyond specialized competences) - can and must be performed with patterns which are taken from the general forms of perceiving-, thinking-, and acting-forms of our civilization."

We have tried in making serious with Hentig's demands and to come to a general understanding of mathematics. For this we have approached the required restructuring of sciences for different subdomains of mathematics, which has been shown eminently fruitful as well in teaching as also in research. However critical reactions and discussions have shown that the reconstruction should be seen in the frame of the more extensive concept of "*general science*". What is to understand under "general science", I have tried to carry out in a contribution for the THD-lecture sequence "Responsibility in the sciences" (1987). [Wi88] Thereafter all efforts belong to the general science which try to disclose sciences and to make them accessible, so that the general researchers can critically explain the possible consequences and outcomes of scientific doings. General science is therefore not an autonomous part of science, but as part of any scientific disci-

pline and also subdiscipline. “*General mathematics*” names the part of general science which is relevant for mathematics. Their efforts aim also in particular to answers concerning the question what mathematics can and should mean for the general public. In the sense of general science, general mathematics is characterized by * the *attitude* to open mathematics for the generality and to make it principally learnable and criticizable, * the *presentation* of mathematical developments in its sense giving, meanings and conditions, * the *imparting* of mathematics in its lifeworld connection even across the borders, * the *explanation* about destinations, procedures, conceptual values, and legitimate validities of mathematics.

Since almost twenty years we discussed the theme “general mathematics” in seminars, colloquia and on meetings. Already 1991 Knut Radbruch has pointed out in a lecture [Ra91] at the Darmstadt seminar “general mathematics” as *task of the general mathematics* the analysis of conditions and realizations of the mathematical; for this he formulated the following: “Since the mathematical is the actual object of research of general mathematics, such a mathematical must also try hard about explicit and implicit traces of that mathematical in all regions of rational reason and cultural forms. An analysis of applications and situations of applications of concrete mathematics can - but must not - show the way to such traces of the mathematical.”

The Darmstadt Seminar has received interestedly Radbruch’s proposal and discussed comprehensively Heidegger’s understanding of the mathematical and tried first of all to trace out the *everyday thinking* ([Le97],[LP97]). An important method for it was to examine *general actions of thinking* on their formal part and its possible abstractions in conventionalized mathematics as for instance:

abstracting concritizing mathematizing simplifying
 allocating counting modelizing synthesizing
 analysing differentiating ordering systematizing
 arranging formalizing presenting tabulating
 categorizing generalizing reasoning visualizing
 classifying idealizing schematizing tabulating
 combining illustrating specializing transforming
 comparing interpretating structurizing typifying

For this, a reader [SAM96] was put together with the presented list of thinking actions which may take to the listed words respectively more descriptions of meanings from different restructurings, to get in this way a possibly broad foundation for tracing out the mathematical: Helpful for the examination of thinking actions were the long-standing experiences in “*restructuring*” of mathematical theories in the sense of general mathematics, which has been performed most

intensively in the fields of linear algebra, mathematical logic and mathematical order and lattice theory. ([Wi81],[Wi82],[PW86],[Wi87],[Wi96a],[Wi97],[Pr98])

The efforts concerning general algebra, in particular about the restructuring of parts of mathematics, have led out on manifold modes to connections be-yound mathematics. In this way we have executed in the frame of formal concept analysis and conceptual knowledge processing already more than 200 projects of applications in cooperation with experts out of varying research fields. This has us not only brought “application projects of concrete mathematics” for analysing the mathematical mentioned by Knut Radbruch, but - which was even more revealing - the possibility to recognize *the mathematical and the mathematics in the thinking of non-mathematicians* during the concrete work on tasks and problems which are important for them. We have generally made the experiment that serious difficulties with mathematics have its ground less in lacking knowledge about mathematical concepts, results and procedures, but in the missing confidence with the *self-evident truth*, *conventionalisms* and *convictions* which are used today by mathematicians in their inner-subjective communication. Since this intimacy cannot directly be imparted, it arises for us sustainedly the basic question: with which educational processes can we reach an appropriate relation of *inworld and mathematics*?

For a self-understanding of todays mathematicians it shall be extensively named a *deficit* which the non-mathematicians usually have, namely about the self-understanding that *sets* in the modern mathematics is generally seen and used as something unquestional existent, that even the question concerning the existence of assumed objects is positively decided in the rule by the return to suitable sets. How little this self-understanding exceeds over the society of mathematicians, we experience every year with the freshmen that the written set thinking is for them not at all well-acquainted. The schools arrange still the picture of mathematics of the 19th century, whose central objects of thinking are still numbers and geometric figures. In this way one has in our community no difficulties with a judgment such as “three and four is seven” and is also not asked: “do you mean with it apples or houses?” A number such as 3 or 347 is accepted as a meaningful object of thinking, and in a relation to objects whose quantity is counted with it, this must be not named or only as well assumed. But if one says: “Think a set inside of you and than the set of all its subsets!”, then one usually releases a lack of understanding, at the best it may be inquired what do you conclude about the total set and their subsets. It is interesting that one hardly gains difficulties with the request: “Think about a tree with all its branches!” or even: “Draw this tree!”.

For the freshmen of mathematics it approximately lasts one or two years until they have taken over the *self-understanding and conventionalisms as the foundation of mathematics*. How this happens in detail and how it can be well-aimed supported, this is considerably a riddle. Perhaps the acclimatization succeeds in set thinking more easily for younger students, when there are given set theoretical insides already on the level of schools. Sets are nothing else than concept extents of which the concept intents are forgotten (as numbers are quantities

which do not pay attention to qualities of numbers). In any case it would be a great win for our society, when with the lifeworld thinking of concepts as its mathematical part, the thinking of sets would be more self-evident.

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footnote ¹

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